Procedure for the Characterisation of Piezoelectric Samples in Non-standard Resonant Modes

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Abstract

The standard characterisation of piezoelectric materials requires specific boundary conditions. However the piezoelectric devices require quite different shaped materials and contour conditions from those that standard imposes. For these reasons, device characterisation usually requires models of equivalent electric circuits for non-electric properties as mechanical losses or coupling factors. Using a piezoelectric resonator model, similar to that used in the standard, we can make a more accurate characterisation of piezoelectric devices. © 1999 Elsevier Science Limited. All rights reserved

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1 Introduction

A piezoelectric element is a resonant cavity under mechanical or electrical stimuli. Its characteristic parameters can be studied from its behaviour in resonant conditions. This behaviour depends on:

- Geometrical factors, as the shape of the element. It implies the use of Cartesian, cylindrical, or spherical coordinates and the corresponding solutions to the differential equations become quite different.
- Mechanical boundary conditions as the surfaces are interfaces between two media with different mechanical properties.
- Electric contour conditions, it means open or short circuit electrodes. It includes the electromechanical coupling since mechanical

waves can be parallel or perpendicular to electric field, and then the electrodes can be over standing or vibrating surfaces.

The aim of the material characterisation is obtaining the physical parameters independent of the shape and boundary conditions.

The one-dimensional model of the piezoelectric resonator proposed by Mason¹ can be applied in most cases but it requires a strict control of the shape parameters and strong dimensional relations to prevent the coupling between different resonant modes. These conditions are taken into account in the standard characterisation² of materials. Then this method is not allowed for certain piezoelectric element geometries used in many device applications.

On other hand some kind of materials (as composites, porous media, thick films deposit over several substrates, etc.) can exhibit variations in their allowed resonant modes³ due to the coupling between the different phases conforming it.

The usual way of characterising devices in piezoelectric applications consists of finding an equivalent electric circuit that reproduces the behaviour of the element. But, this assumption is neither simple nor accurate in all cases and the obtained parameters—quality factors, figures of merit—have no relevant meaning out of this specific application.

Then we propose the use of an improved material characterisation technique to get the device parameters in resonant conditions. The set of relevant parameters includes one dielectric permittivity constant, one piezoelectric constant, and one or two elastic constants. Moreover, this procedure takes into account the electrical and mechanical losses and the imperfect electromechanical energy conversions.⁴ To do this it uses complex parameters for a complete description of the device behaviour, as

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is made for materials according to the method proposed by Smits⁵ and following our own previous works.^{6,7} Of course, the set of parameters is not exactly that corresponding to the materials, but we can use a similar notation to denote similar properties.

2 Preliminary Steps

Using the above mentioned Mason's Model we can choose a minor of elasto-piezo-dielectric characteristic matrix that includes only the parameters involved in an specific one-dimensional resonant mode. We can use then a set of two linear equations instead of the whole matricial set in order to obtain the function describing the admittance or impedance of the piezoelectric element. Then using techniques of complex impedance spectroscopy, we will get the required data to do the characterisation.

The shape of the sample is very important in order to select the simplest system of coordinates in order to find the electrical immittance of the element. Standing waves into a parallelepipedic resonant cavity can be studied by means of Cartesian coordinates, and their wave functions involve trigonometric expressions. But standing waves appearing in a cylindrical or spherical cavity sometimes must be expressed by means of wave functions that involve more complicated functions as modified quotients of Bessel's functions.⁸

The other important point to take into account is that relative to the boundary conditions.

Mechanically, we can find two extreme situations: traction-free or displacement-free surfaces according to the well known open or closed ends conditions for the standing waves. For the electric behaviour, open or short circuit conditions in electrodes are not only determined by external stimuli but also by the relative direction between mechanical wave and electric field. This is because the electroded surfaces can be clamped or free modifying their electrical response.

3 Improving the Procedure

In order to simplify the following steps we will assume that we can use Cartesian coordinates because of the shape of the piezoelectric element. For every resonant mode in devices or simply nonstandard modes in a material sample there should be chosen one of the four possibilities for the constitutive equations, according, of course, with the preliminary step described above. For each couple of constitutive equations, we obtain a set of different parameters—one dielectric, one elastic, and one piezoelectric. For example, on a thin plate with their major surfaces fully covered by electrodes and vibrating in the higher fundamental mode, we will have mechanically clamped conditions and the voltage between electrodes driven by the external stimulation and giving an electrical field parallel to the acoustic wave direction. Then we will use

$$T = c^*S - h^*D$$

 $E = -h^*S + (\varepsilon_0 \varepsilon^*)^{-1}D$

where c^* is the elastic stiffness, ε^* the relative dielectric permittivity, and h^* the piezoelectric constant, as corresponding to thickness extensional mode in standard characterisation. The main difference is that now the plate may have applied some mechanical restrictions (e.g. can be fixed or clamped to any substrate, or be placed into a resonant cavity, etc.).

Once the correct set of constitutive equation is taken, we find the expression that gives the electric impedance or the admittance of the element. Admittance is the better election for the characterisation in modes where the involved dielectric constant is 'free', whereas the impedance will be taken for those modes including the 'clamped' dielectric constant.

As a result of this technique, we obtain a set of constants with the same name, and physical



Fig. 1. Schematic view of ultrasonic piezoelectric buzzer.

 Table 1. characterisation of an ultrasonic piezoelectric buzzer element in the 10 MHz band

Ceramic density Thickness Electrode surface	$7.5 \mathrm{gr}\mathrm{cm}^{-3}$ $0.03 \mathrm{cm}$ $1.267 \mathrm{cm}^{2}$	
Experimental data		
Resonance band Antiresonance band	f_s (MHz): 9.375 $Z_{reson}(\Omega)$: 0.2575–0.0103i f_p (MHz): 10.575 $Z_{ares}(\Omega)$: 16.227–1.375i	Q_{reson} : 52 Q_{ares} : 49
Results		
Coupling factor Stiffness Piezoelectric constant dielectric permittivity	$k_t(\%): 49.9-1.9i$ c^* (GNM ⁻²): 301.8+6.0i h^* (MVm ⁻¹):1831+35i ε^* (relative):2522-234i	Q_{mec} : 50 Q_{piezo} : 53 Q_{elec} : 11

meaning, as those used in material characterisation although they are not characteristic of the material. This is because they differ from a sample to another due to the external restrictions.

4 Example and Conclusions

In this work, we choose a commercial ultrasonic piezoelectric buzzer. The piezoelectric active material is a PZT ceramic shaped as thin disk and glued to a wider metallic disk. This one is joined and in contact only in its border to a metallic shell that creates a resonant cavity, as shown in Fig. 1. For the 10 MHz resonant band we are in similar conditions to thickness resonant mode of thin plates. This band corresponds to the highest fundamental resonance and then the dielectric permittivity above the antiresonance frequency is the clamped one. As for the thickness resonant mode, we find the first overtone of the antiresonance band 3 times the fundamental antiresonance frequency.



Fig. 2. Graph for the admittance versus frequency. Experimental (points) and calculated (line) curves are plotted.



Fig. 3. Graph for permittivity, imaginary versus real part. Experimental (points) and calculated (line) curves are plotted.

We will use then, for the characterisation in the 10 MHz band, the same procedure as for the thickness extensional mode of plates as described in a previous work.⁷ We obtain results in Table 1. Using the elastic, piezoelectric and dielectric constants (Table 1), we calculated the admittance of the element for frequencies between 5 and 15 MHz. We show the calculated curve in the same graph (Fig. 2) that contains the experimental data in order to compare the real and the predicted behaviour. The same strategy is followed for the real and imaginary parts of the dielectric permittivity, and in the same range of frequencies (Fig. 3).

As can be seen, the accuracy in predictions is similar to that obtained from samples following strictly the standard conditions. The obtained constants cannot be taken as characteristic material constants. However, they can be used to predict the behaviour of the piezoelectric element under electrical stimulation in conditions under which the element is commonly used.

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